

自動計算システムによる輻射補正計算における スレプトンの非線形ゲージ固定項の有用性

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[arXiv:1006.3491 \(hep-ph\)](https://arxiv.org/abs/1006.3491); PoS (CPP2010) 005

1. はじめに

• Feynman 振幅の自動計算

 LHC, ILC等の加速器実験のシミュレーションでは
必須アイテム

- 終状態は多体
- 様々な未発見の新粒子が预言されている
- 摂動の高次の効果の計算には膨大な数のダイアグラム

• 自動計算システム

GRACE	Prog. Theor. Phys. Suppl. 138 (2000) 18 Comput. Phys. Commun. 153 (2003) 106
CompHEP	Nucl. Instrum. Meth. A534 (2004) 250
CalcHEP	hep-ph/0412191
FeynArts/FormCalc	Comput. Phys. Commun. 140 (2001) 418; 143 (2002) 54
MadGraph	JHEP 0302 (2003) 027
SloopS	Phys. Lett. B660 (2008) 550

2. GRACEシステム

	SM	MSSM
Tree level	GRACE	GRACE/SUSY
	http://minami-home.kek.jp/	
1-loop level	GRACE-loop	GRACE/SUSY-loop

(計算法)

	SM	MSSM
Tree level	GRACE	GRACE/SUSY
	ヘリシティ振幅 (CHANEL)	
1-loop level	GRACE-loop	GRACE/SUSY-loop
	行列要素	
	(REDUCE, Form)	(REDUCE)

(ゲージ不変性検査)

	SM	MSSM
Tree level	GRACE	GRACE/SUSY
共変ゲージ・ユニタリゲージ間で比較		
1-loop level	GRACE-loop	GRACE/SUSY-loop
非線形ゲージを導入		

何故，非線形ゲージが必要なのか？

- 一般に， R_ξ -ゲージではゲージボソンの伝搬関数

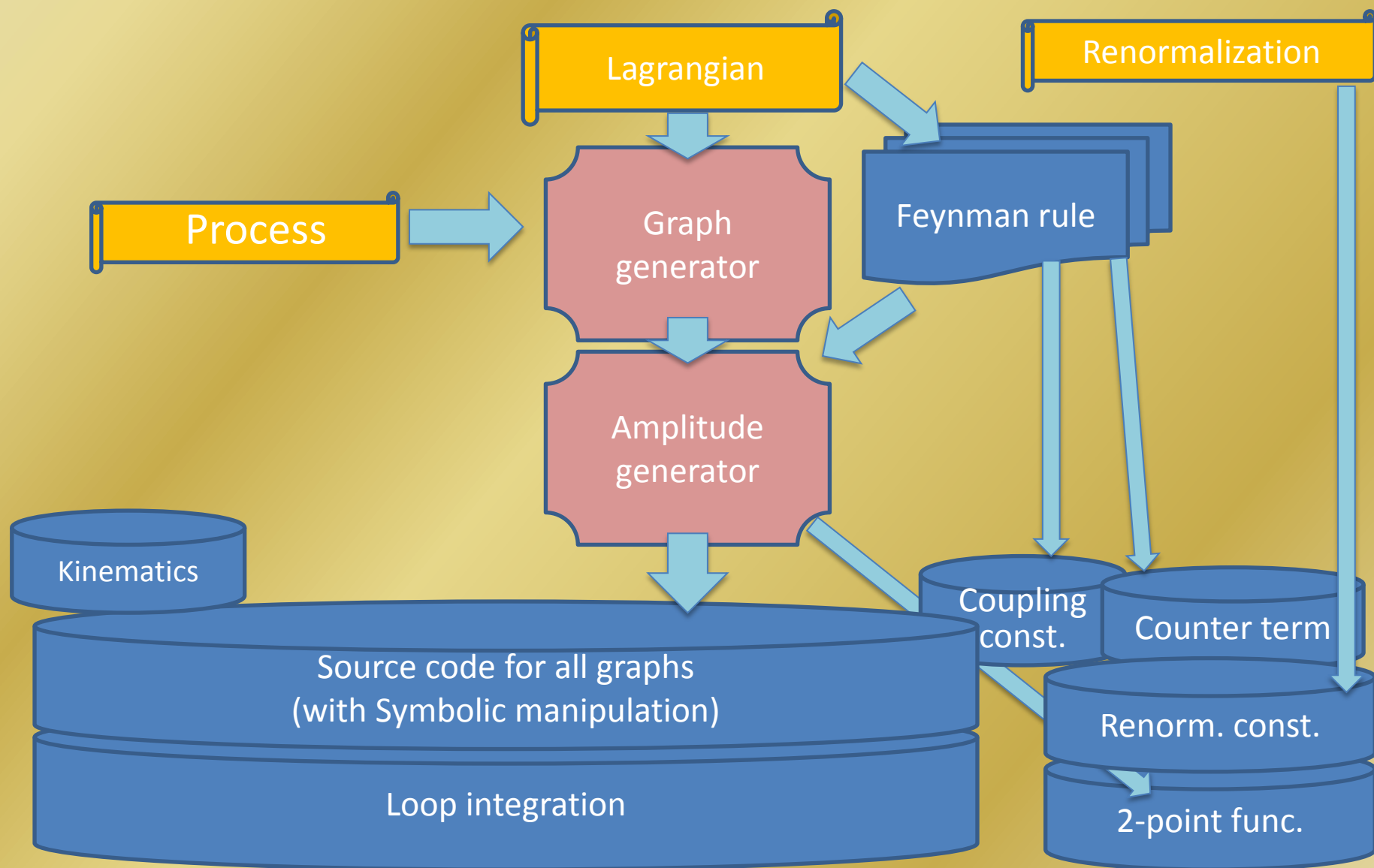
(例えば $\frac{1}{k^2 - M_W^2 + i\varepsilon} \left(g_{\mu\nu} - (1 - \xi_W) \frac{k_\mu k_\nu}{k^2 - \xi_W M_W^2} \right)$) の

$k_\mu k_\nu$ 項が，ループ計算では数値的不安定性をもたらす

- 't Hooft-Feynmanゲージ ($\xi_W = 1$) の範囲で動かせるパラメータを導入してゲージ不変性チェックをしたい

➡ 非線形ゲージ(以下，NLGと略称)

システム構成図



3. GRACE/SUSY-loopの繰り込み処方

- 繰り込みの方針

- On mass-shell繰り込み

例えばWボソンの伝搬関数の極 M_W^2 に対して

$$\text{Re } \hat{\Pi}_W(M_W^2) = 0$$

ここで, $\hat{\Pi}_W(q^2) = \Pi_W(q^2) + \delta M_W^2 - \delta Z_W(q^2 - M_W^2)$

- ゲージ不変

NLGの導入

- 反応過程に依らない

cf. N. Baro, F. Boudjema, A. Semenov, Phys. Rev. D78 (2008) 115003.

● On mass-shell繰り込み

⇒ On mass-shell繰り込みの条件を次の粒子に課す

➤ ゲージボソン

➤ 通常のフェルミオン(クォーク, レプトン)

➤ スフェルミオン

➤ ヒッグスボソン(A^0 , H^0)

注) h^0 には課さない ← 大きな質量補正が必要

➤ 最も軽いニュートラリーノ($\tilde{\chi}_1^0$)

➤ チャージノーノ($\tilde{\chi}_1^+$, $\tilde{\chi}_2^+$)

* 繰り込み条件の数は独立な繰り込み定数の数

●ゲージ固定項

$$L_{\text{gf}} = -\frac{1}{\xi_W} |F_{W^+}|^2 - \frac{1}{2\xi_Z} (F_Z)^2 - \frac{1}{2\xi_\gamma} (F_\gamma)^2$$

$$F_{W^+} = (\partial_\mu + ie\tilde{\alpha}A_\mu + igc_W\tilde{\beta}Z_\mu)W^{+\mu} + i\xi_W \frac{g}{2} (v + \tilde{\delta}_h h^0 + \tilde{\delta}_H H^0 + i\tilde{\kappa}G^0)G^+$$

$$F_{W^-} = (\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^{-\mu} - i\xi_W \frac{g}{2} (v + \tilde{\delta}_h h^0 + \tilde{\delta}_H H^0 - i\tilde{\kappa}G^0)G^-$$

$$F_Z = \partial_\mu Z^\mu + \xi_Z \frac{g_Z}{2} (v + \tilde{\varepsilon}_h h^0 + \tilde{\varepsilon}_H H^0)G^0 \quad F_\gamma = \partial_\mu A^\mu$$

$$v = \sqrt{v_1^2 + v_2^2} \quad M_W = \frac{g v}{2} \quad M_Z = \frac{g_Z v}{2}$$

NLGパラメータ $(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}_h, \tilde{\delta}_H, \tilde{\kappa}, \tilde{\varepsilon}_h, \tilde{\varepsilon}_H)$

SM: G. Bélanger et al., Phys. Rep. 430 (2006) 117

MSSM: J. Fujimoto et al., Nucl. Phys. (Proc. Suppl.) 157 (2006) 157; Phys. Rev. D75 (2007) 113002

●スレプトンに関する繰り込み

➤1世代当たり, 3個の質量繰り込み定数, 5個の波動関数繰り込み定数, 1個の混合角繰り込み定数がある

$$\delta m_{\tilde{l}_1}, \delta m_{\tilde{l}_2}, \delta m_{\tilde{\nu}_\ell}, \delta Z_{\tilde{l}_i \tilde{l}_j} (i, j = 1, 2), \delta Z_{\tilde{\nu}_\ell}, \delta \theta_\ell$$

➤次の繰り込み条件を課す (第3世代について表記)

1. On mass-shell条件

$$\delta m_{\tilde{\ell}}^2 = -\Sigma_{\tilde{\ell} \tilde{\ell}}(m_{\tilde{\ell}}^2), \quad \tilde{\ell} = \tilde{\tau}_1, \tilde{\tau}_2, \tilde{\nu}_\tau$$

2. Residue条件

$$\delta Z_{\tilde{\tau}_i \tilde{\tau}_i} = \frac{\partial}{\partial q^2} \Sigma_{\tilde{\tau}_i \tilde{\tau}_i}(q^2) \Big|_{q^2 \rightarrow m_{\tilde{\tau}_i}^2} \equiv \Sigma'_{\tilde{\tau}_i \tilde{\tau}_i}(m_{\tilde{\tau}_i}^2)$$

$$\delta Z_{\tilde{\nu}_\tau} = \Sigma'_{\tilde{\nu}_\tau \tilde{\nu}_\tau}(m_{\tilde{\nu}_\tau}^2)$$

3. Decoupling条件

$$\frac{1}{2} \delta Z_{\tilde{\tau}_i \tilde{\tau}_j} = -\frac{\Sigma_{\tilde{\tau}_i \tilde{\tau}_j}(m_{\tilde{f}_j}^2)}{m_{\tilde{\tau}_i}^2 - m_{\tilde{\tau}_j}^2}, \quad i \neq j$$

4. SU(2)関係式

$$\delta \theta_\tau = [\delta m_{\tilde{\nu}_\tau}^2 - \delta(M_W^2 \cos 2\beta - m_\tau^2) - \cos^2 \theta_\tau \delta m_{\tilde{\tau}_1}^2 - \sin^2 \theta_\tau \delta m_{\tilde{\tau}_2}^2] \times \frac{1}{\sin 2\theta_\tau (m_{\tilde{\tau}_2}^2 - m_{\tilde{\tau}_1}^2)}$$

4. NLGの拡張

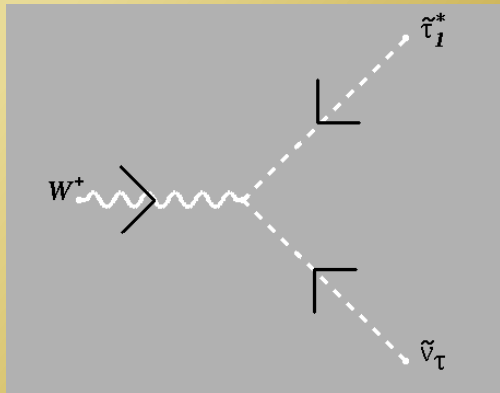
スレプトンの2次形式を加える

$$F_{W^+} = (\partial_\mu + ie\tilde{\alpha}A_\mu + igc_W\tilde{\beta}Z_\mu)W^{+\mu} + i\xi_W \frac{g}{2} (v + \tilde{\delta}_h h^0 + \tilde{\delta}_H H^0 + i\tilde{\kappa}G^0)G^+ \\ + i\xi_W g \sum_{i=1,2} \left\{ \tilde{c}_i^e (\tilde{e}_i^* \tilde{\nu}_e) + \tilde{c}_i^\mu (\tilde{\mu}_i^* \tilde{\nu}_\mu) + \tilde{c}_i^\tau (\tilde{\tau}_i^* \tilde{\nu}_\tau) \right\}$$

$$F_{W^-} = (\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^{-\mu} - i\xi_W \frac{g}{2} (v + \tilde{\delta}_h h^0 + \tilde{\delta}_H H^0 - i\tilde{\kappa}G^0)G^- \\ - i\xi_W g \sum_{i=1,2} \left\{ \tilde{c}_i^e (\tilde{\nu}_e^* \tilde{e}_i) + \tilde{c}_i^\mu (\tilde{\nu}_\mu^* \tilde{\mu}_i) + \tilde{c}_i^\tau (\tilde{\nu}_\tau^* \tilde{\tau}_i) \right\}$$

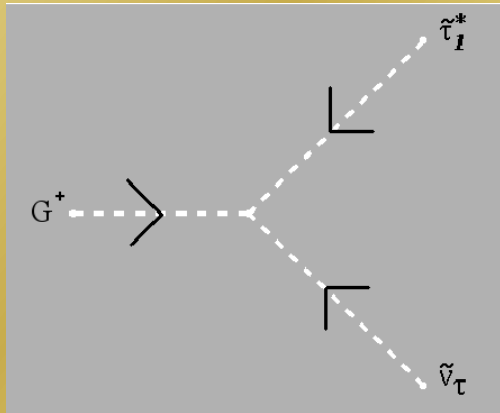
NLGパラメータ $(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}_h, \tilde{\delta}_H, \tilde{\kappa}, \tilde{\varepsilon}_h, \tilde{\varepsilon}_H, \tilde{c}_i^e, \tilde{c}_i^\mu, \tilde{c}_i^\tau)$

● Feynman ルール ($\xi_W = 1$, \tilde{c}_1^τ に対して)



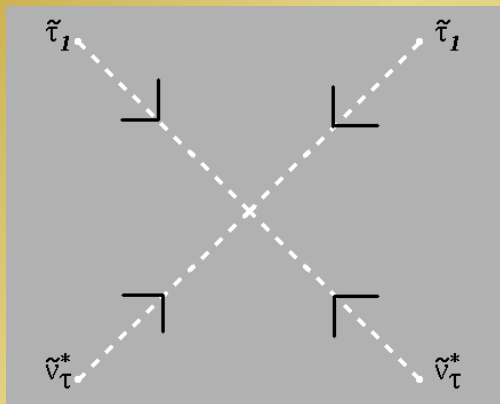
$$i \frac{g}{\sqrt{2}} \cos \theta_\tau (p_\mu^{\tilde{\nu}} - p_\mu^{\tilde{\tau}}) \quad -ig\tilde{c}_1^\tau (p_\mu^{\tilde{\nu}} + p_\mu^{\tilde{\tau}})$$

線形ゲージ + NLG



$$i \frac{g}{\sqrt{2}M_W} \cos \theta_\tau (M_W^2 \cos 2\beta - m_\tau^2) \quad -igM_W \tilde{c}_1^\tau$$

線形ゲージ + NLG



$$-i \frac{g_Z^2}{4} \cos^2 \theta_\tau + i \frac{1}{2} \sin^2 \theta_\tau \left(g_Z^2 \sin^2 \theta_W - \frac{g^2 m_\tau^2}{M_W^2 \cos^2 \beta} \right)$$

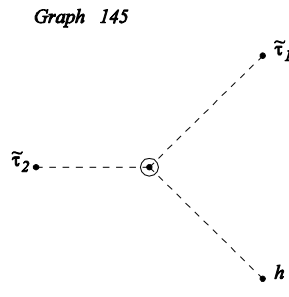
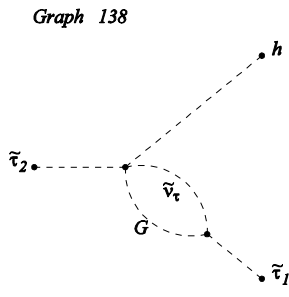
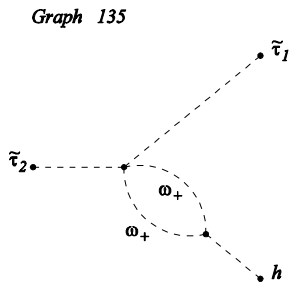
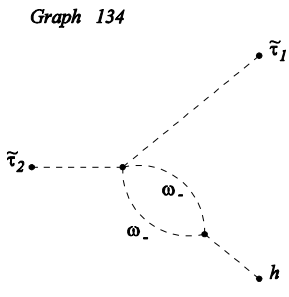
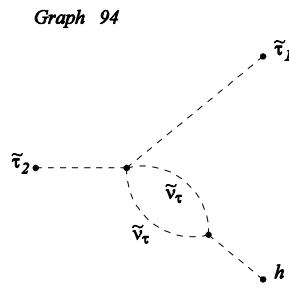
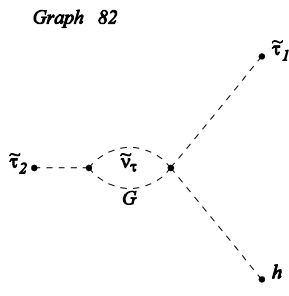
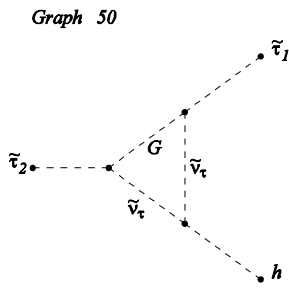
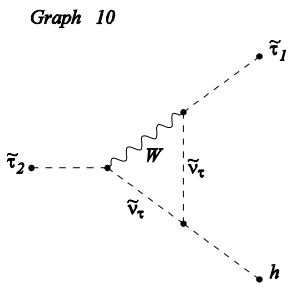
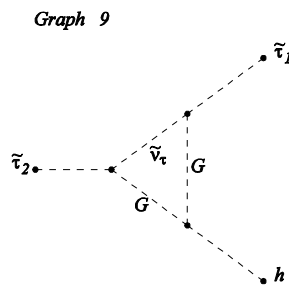
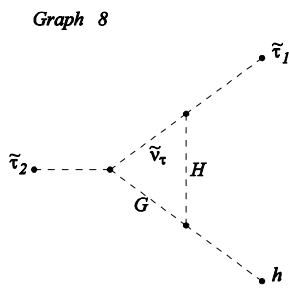
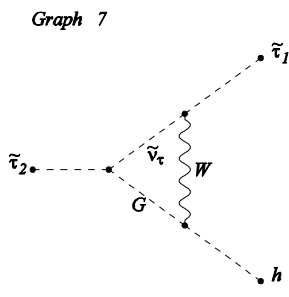
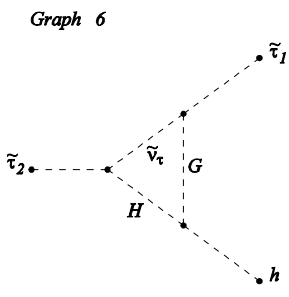
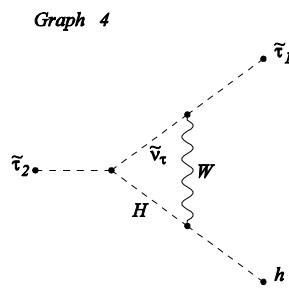
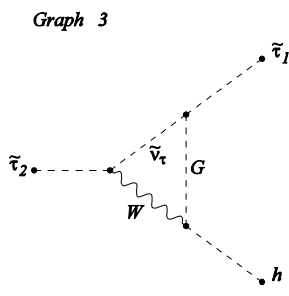
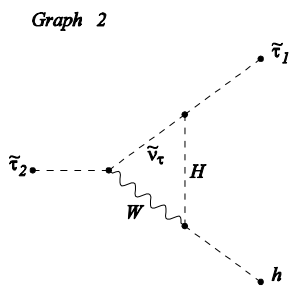
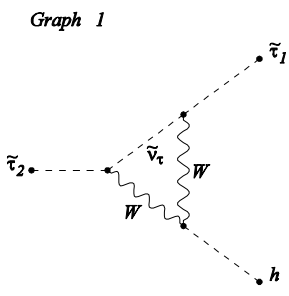
線形ゲージ +

$$-ig^2 (\tilde{c}_1^\tau)^2 \quad \text{NLG}$$

●数値的な検査の一例

$$\tilde{\tau}_2^- \rightarrow \tilde{\tau}_1^- + h^0$$

Loop + CT
151 diagrams



graph	UV part			Finite part	
	$(\tilde{c}_1^r)^0$	$(\tilde{c}_1^r)^1$	$(\tilde{c}_1^r)^2$	$(\tilde{c}_1^r)^1$	$(\tilde{c}_1^r)^2$
	Virtual				
1	-2.864048E+02	8.100752E+02	-5.728097E+02	-6.902655E+00	6.310783E+00
2	2.640140E+02	-3.733722E+02	0.000000E+00	4.340137E+00	0.000000E+00
3	1.281803E+03	-2.015262E+03	2.864048E+02	2.219785E+01	-3.032220E+00
4	-2.459420E+02	3.478145E+02	0.000000E+00	-4.165453E+00	0.000000E+00
6	0.000000E+00	0.000000E+00	0.000000E+00	2.651421E-01	0.000000E+00
7	-1.470710E+03	1.877379E+03	2.864048E+02	-2.268075E+01	-3.192840E+00
8	0.000000E+00	0.000000E+00	0.000000E+00	-3.626202E-01	0.000000E+00
9	0.000000E+00	0.000000E+00	0.000000E+00	-7.996177E-02	-8.572352E-02
10	1.839419E+02	-5.202662E+02	3.678837E+02	5.671871E+00	-4.659912E+00
50	0.000000E+00	0.000000E+00	0.000000E+00	1.638727E-02	1.756807E-02
82	1.467649E+03	2.020973E+02	0.000000E+00	-2.152933E+00	0.000000E+00
94	-9.034420E+01	0.000000E+00	-3.678837E+02	0.000000E+00	4.642344E+00
134	0.000000E+00	-2.025188E+02	0.000000E+00	1.726551E+00	0.000000E+00
135	0.000000E+00	-2.025188E+02	0.000000E+00	1.726551E+00	0.000000E+00
138	-1.279135E+03	2.020973E+02	0.000000E+00	-2.266272E+00	0.000000E+00
	Counter Term				
145	5.330665E+02	-1.255253E+02	0.000000E+00	2.666161E+00	0.000000E+00
	Total				
	-6.032040E-20	5.714153E-27	1.019217E-29	-3.995728E-23	-2.514610E-31

●スレプトンの繰り込み条件を変えた場合

- Residue条件はスレプトン-2には課さない

$$\delta Z_{\tilde{\tau}_1 \tilde{\tau}_1} = \Sigma'_{\tilde{\tau}_1 \tilde{\tau}_1} (m_{\tilde{\tau}_1}^2)$$

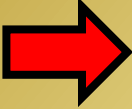
$$\delta Z_{\tilde{\tau}_2 \tilde{\tau}_2} = \delta Z_{\tilde{\tau}_1 \tilde{\tau}_1} + \frac{\cos 2\theta_\tau}{\sin 2\theta_\tau} (\delta Z_{\tilde{\tau}_1 \tilde{\tau}_2} + \delta Z_{\tilde{\tau}_2 \tilde{\tau}_1})$$

- SU(2) 関係式は使わない

$$\delta\theta_\tau = \frac{1}{4} (\delta Z_{\tilde{\tau}_1 \tilde{\tau}_2} - \delta Z_{\tilde{\tau}_2 \tilde{\tau}_1})$$

[Guaschの関係式:

Phys. Lett. B437 (1998) 88]

 NLG検査のFinite partで $(\tilde{c}_1^\tau)^1$ 項が
-0.1290782e+01 となって消えのこる

5. まとめ

- **LHC, ILC**等の加速器実験のシミュレーションでは**Feynman**振幅の自動計算システムは必須であるが、数値計算の結果を検査するための道具立てが必要となる
- 自動計算システムによるループ計算でゲージ不変性検査を行うには非線形ゲージ固定項を導入する必要がある
- 非線形ゲージパラメータによる検査で、どのような繰り込み条件が妥当かを確かめることが出来る


Backup

BRST 変換

$$\begin{aligned}\delta_{\text{BRST}} \tilde{v}_\tau &= -\frac{i}{2} \theta \left(\sqrt{2} g \omega_+ \tilde{\tau}_L + (g_Z \cos^2 \theta_W \omega_Z + e \omega_\gamma) \tilde{v}_\tau \right) + \frac{i}{2} \theta (-g_Z \sin^2 \theta_W \omega_Z + e \omega_\gamma) \tilde{v}_\tau \\ &= -\frac{i}{2} \theta \left(\sqrt{2} g \omega_+ \tilde{\tau}_L + g_Z \omega_Z \tilde{v}_\tau \right)\end{aligned}$$

$$\delta_{\text{BRST}} \tilde{\tau}_L = -\frac{i}{2} \theta \left(\sqrt{2} g \omega_- \tilde{v}_\tau - (g_Z \cos^2 \theta_W \omega_Z + e \omega_\gamma) \tilde{\tau}_L \right) + \frac{i}{2} \theta (-g_Z \sin^2 \theta_W \omega_Z + e \omega_\gamma) \tilde{\tau}_L$$

$$\delta_{\text{BRST}} \tilde{\tau}_R = +i \theta (-g_Z \sin^2 \theta_W \omega_Z + e \omega_\gamma) \tilde{\tau}_R$$

$$\tilde{\tau}_1 = \cos \theta_\tau \tilde{\tau}_L + \sin \theta_\tau \tilde{\tau}_R$$


$$\delta_{\text{BRST}} \tilde{\tau}_1 = -\frac{i}{2} \theta \left(\sqrt{2} g \cos \theta_\tau \omega_- \tilde{v}_\tau - g_Z \cos \theta_\tau \omega_Z (\cos \theta_\tau \tilde{\tau}_1 - \sin \theta_\tau \tilde{\tau}_2) + 2(g_Z \sin^2 \theta_W \omega_Z - e \omega_\gamma) \tilde{\tau}_1 \right)$$

Additional terms of ghost Lagrangian ($\xi_W = 1$)

$$L_{\text{ghost}}^{\text{NLG}} = -\bar{\omega}_+ \theta^{-1} \delta_{\text{BRST}} \left(i g \tilde{c}_1^\tau \tilde{\tau}_1^* \tilde{v}_\tau \right) + h.c.$$

$$\begin{aligned}&= \frac{g}{2} \tilde{c}_1^\tau \left(\sqrt{2} g \cos \theta_\tau \omega_+ \tilde{v}_\tau^* - g_Z \cos \theta_\tau \omega_Z (\cos \theta_\tau \tilde{\tau}_1^* - \sin \theta_\tau \tilde{\tau}_2^*) + 2(g_Z \sin^2 \theta_W \omega_Z - e \omega_\gamma) \tilde{\tau}_1^* \right) \tilde{v}_\tau \\ &\quad - \frac{g}{2} \tilde{c}_1^\tau \tilde{\tau}_1^* \left(\sqrt{2} g \omega_+ (\cos \theta_\tau \tilde{\tau}_1 - \sin \theta_\tau \tilde{\tau}_2) + g_Z \omega_Z \tilde{v}_\tau \right) + h.c.\end{aligned}$$